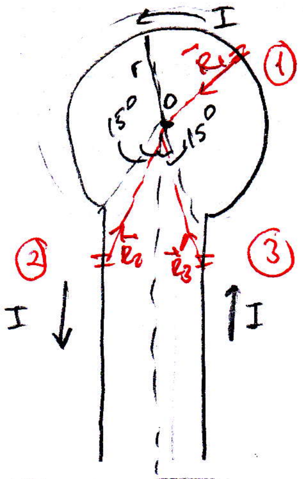
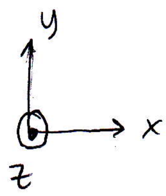
Q

An incomplete circular loop with very long leads carries a current of $I = 10$ [A], as shown in figure. Calculate the magnetic field intensity and magnetic flux density at the center of the loop.



$$\vec H = \int \frac{I \, d\vec l \times \vec R}{4\pi |\vec R|^3}$$



A ① $d\vec l_1 = \rho \, d\phi \, \vec e_\phi \qquad \vec R_1 = -r \, \vec e_\rho \qquad d\vec l_1 \times \vec R_1 = \rho r \, d\phi \, \vec e_z$

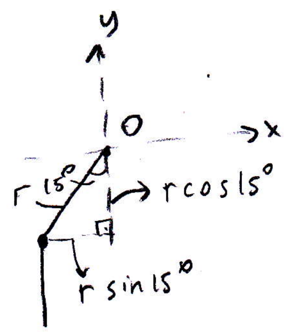
$$\vec H_1 = \int \frac{I \rho r \, d\phi}{4\pi r^3} \vec e_z = \frac{I}{4\pi r} \int_{\phi = \pi/12}^{23\pi/12} d\phi \, \vec e_z = \frac{I}{4\cancel{\pi} r} \frac{11\cancel{\pi}}{6} \vec e_z = \frac{11 I}{24 r} \vec e_z$$

$\rho = r$

② $d\vec l_2 = -dy \, \vec e_y \qquad \vec R_2 = x \, \vec e_x + y \, \vec e_y \qquad d\vec l_2 \times \vec R_2 = x \, dy \, \vec e_z$

$$\vec H_2 = \int \frac{I x \, dy}{4\pi (x^2+y^2)^{3/2}} \vec e_z$$

$$\vec H_2 = \frac{-I r \sin 15°}{4\pi} \int_{y=-r\cos 15°}^{\infty} \frac{dy}{(x^2+y^2)^{3/2}} \vec e_z$$



$$\int \frac{dt}{(t^2+c^2)^{3/2}} = \frac{1}{c^2} \frac{t}{\sqrt{t^2+c^2}} \Rightarrow \int_{y=-r\cos 15°}^{-\infty} \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{\sqrt{x^2+y^2}} \Bigg|_{y=-r\cos 15°}^{-\infty}$$

$$= 0 - \frac{1}{x^2}\left(-1 - \frac{(-r\cos 15°)}{r}\right)$$

$$= -\frac{1}{r^2 \sin^2 15°}\left(-1 + \cos 15°\right)$$

$$\vec{H}_2 = \frac{-I\,\cancel{r\sin 15^\circ}}{4\pi} \cdot \frac{-(-1+\cos 15^\circ)}{r^{\cancel{2}}\sin^{\cancel{2}} 15^\circ}\,\vec{e_z} = \frac{I(\cos 15^\circ - 1)}{4\pi r \sin 15^\circ}\,\vec{e_z}$$

③ $\vec{dl_3} = dy\,\vec{e_y} \quad \vec{R_3} = -x\,\vec{e_x} + y\,\vec{e_y} \quad \vec{dl_3}\times\vec{R_3} = x\,dy\,\vec{e_z}$

$$\vec{H}_3 = \frac{Ix}{4\pi}\int_{y=-\infty}^{-r\cos 15^\circ} \frac{dy}{(x^2+y^2)^{3/2}}\,\vec{e_z} = \frac{Ix}{4\pi x^2}\left(\left.\frac{y}{\sqrt{x^2+y^2}}\right|_{y=-\infty}^{-r\cos 15^\circ}\right) = \frac{I}{4\pi x}(\cos 15^\circ - 1)\,\vec{e_z}$$

$$\vec{H}_3 = \frac{I(\cos 15^\circ - 1)}{4\pi r \sin 15^\circ}\,\vec{e_z}$$

$$\vec{H} = \vec{H}_1 + \vec{H}_2 + \vec{H}_3$$

$$\vec{H} = \left[\frac{I(\cos 15^\circ - 1)}{2\pi r \sin 15^\circ} + \frac{11 I}{24 r}\right]\vec{e_z}$$

$$\vec{B} = \mu\vec{H} = \left[\frac{\mu I(\cos 15^\circ - 1)}{2\pi r \sin 15^\circ} + \frac{11\mu I}{24 r}\right]\vec{e_z}$$