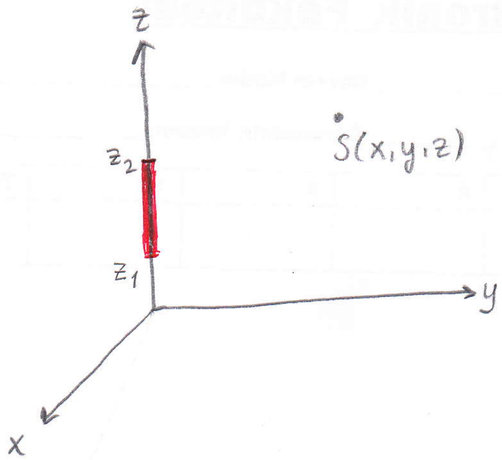
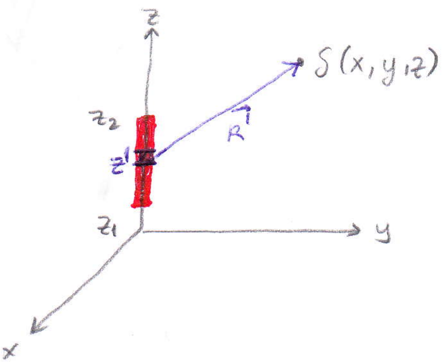
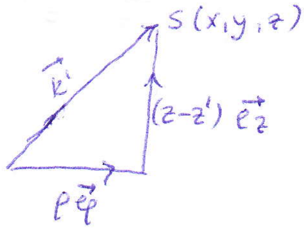
Q



A finite line carrying uniform line charge density λ [C/m]. Calculate electric field $\vec{E}$ at $S(x,y,z)$.

A





$$\vec{E}_S = \frac{1}{4\pi\varepsilon_0}\int \frac{\partial\lambda}{|\vec{R}'|^2}\,\vec{e}_{R'}$$

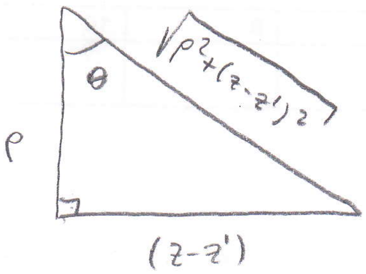
$$\vec{R}' = \rho\,\vec{e_\rho} + (z-z')\,\vec{e_z}$$

$$\vec{e}_{R'} = \frac{\vec{R}'}{|\vec{R}'|} \qquad \partial\lambda = \lambda\, dl = \lambda\, dz'$$

$$\vec{E}_S = \frac{1}{4\pi\varepsilon_0}\int_{z_1}^{z_2} \frac{\lambda\, dz'\left[\rho\,\vec{e_\rho} + (z-z')\vec{e_z}\right]}{\left[\rho^2 + (z-z')^2\right]^{3/2}}$$

$$\vec{E}_S = \frac{\lambda}{4\pi\varepsilon_0}\int_{z_1}^{z_2} \frac{\rho\, dz'}{\left[\rho^2+(z-z')^2\right]^{3/2}}\,\vec{e_\rho} + \frac{\lambda}{4\pi\varepsilon_0}\int_{z_1}^{z_2} \frac{(z-z')\, dz'}{\left[\rho^2+(z-z')^2\right]^{3/2}}\,\vec{e_z}$$

$$A = \int \frac{\rho\, dz'}{\left[\rho^2+(z-z')^2\right]^{3/2}} = \rho \int \frac{dz'}{\left[\rho^2+(z-z')^2\right]^{3/2}}$$



$$\tan\theta = \frac{z-z'}{\rho}$$

$$(z-z') = \rho\tan\theta = \rho\frac{\sin\theta}{\cos\theta}$$

$$-dz' = \rho\left(\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}\right)d\theta$$

$$\boxed{dz' = \frac{-\rho}{\cos^2\theta}\,d\theta}$$

$$A = \rho\int\frac{-\rho\, d\theta/\cos^2\theta}{\left[\rho^2+\rho^2\tan^2\theta\right]^{3/2}} = -\rho^2\int\frac{d\theta}{\cos^2\theta\left[\rho^2\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\right]^{3/2}} = -\rho^2\int\frac{d\theta}{\rho^3/\cos\theta}$$

$$A = -\frac{1}{\rho}\int\cos\theta\, d\theta = -\frac{1}{\rho}\sin\theta = -\frac{1}{\rho}\frac{(z-z')}{\sqrt{\rho^2+(z-z')^2}}$$

$$B = \int\frac{(z-z')\,dz'}{\left[\rho^2+(z-z')^2\right]^{3/2}}$$

$$\rho^2+(z-z')^2 = u$$

$$-2(z-z')\,dz' = du$$

$$(z-z')\,dz' = -\frac{du}{2}$$

$$B = \int\frac{-du}{2u^{3/2}} = \frac{1}{\sqrt{u}} = \frac{1}{\sqrt{\rho^2+(z-z')^2}}$$

$$\vec{E}_s = \frac{\lambda}{4\pi\varepsilon_0\rho} \left. \frac{(z'-z)}{\sqrt{\rho^2+(z-z')^2}} \right|_{z'=z_1}^{z_2} \vec{e_\rho} + \frac{\lambda}{4\pi\varepsilon_0} \left. \frac{1}{\sqrt{\rho^2+(z-z')^2}} \right|_{z'=z_1}^{z_2} \vec{e_z}$$

$$\vec{E}_s = \frac{\lambda}{4\pi\varepsilon_0\rho} \left[\frac{(z_2-z)}{\sqrt{\rho^2+(z-z_2)^2}} - \frac{(z_1-z)}{\sqrt{\rho^2+(z-z_1)^2}} \right] \vec{e_\rho} +$$

$$\frac{\lambda}{4\pi\varepsilon_0} \left[\frac{1}{\sqrt{\rho^2+(z-z_2)^2}} - \frac{1}{\sqrt{\rho^2+(z-z_1)^2}} \right] \vec{e_z}$$

If $z_1 \to -\infty$ and $z_2 \to \infty$

$$\vec{E}_s = \frac{\lambda}{4\pi\varepsilon_0\rho} \underbrace{\left. \frac{z'-z}{|z-z'|} \right|_{z'=-\infty}^{\infty}}_{2} \vec{e_\rho} + \underbrace{\frac{1}{4\pi\varepsilon_0} \left. \frac{1}{|z-z'|} \right|_{z'=-\infty}^{\infty}}_{0} \vec{e_z}$$

$$\vec{E}_s = \frac{\lambda}{2\pi\varepsilon_0\rho} \vec{e_\rho}$$