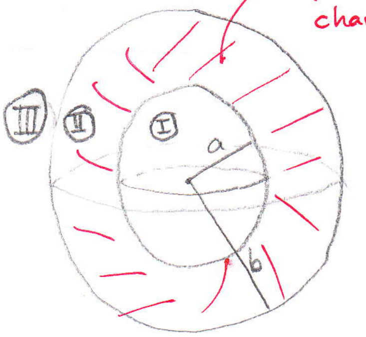
**Q** Suppose there is $\rho_v$ = constant charge density between two concentric spheres as shown in figure. Find electric field intensity in the regions I, II and III.

**A**




Figure

Gauss's law

$$\oint \vec{E} \cdot \vec{ds} = \frac{Q}{\varepsilon}$$

$$\vec{E} = E\,\vec{e_r}$$

**Region I**

$Q = \int \rho_v dv = 0$ there is no charge in region I.

$$\boxed{\vec{E} = 0}$$

**Region II**

$$Q = \int_v \rho_v\, dv = \int_{r=a}^{r}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \rho\, r^2 \sin\theta\, dr\, d\theta\, d\phi = \rho \left.\frac{r^3}{3}\right|_{r=a}^{r} \left.-\cos\theta\right|_{\theta=0}^{\pi} \left.\phi\right|_{\phi=0}^{2\pi}$$

$Q = \frac{4\pi\rho}{3}(r^3 - a^3)$ $a < r < b$ Here $r$ is radius of any sphere in region II.

$$\oint \vec{E} \cdot \vec{ds} = \oint (E\vec{e_r}) \cdot (r^2 \sin\theta\, d\theta\, d\phi\, \vec{e_r}) = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} E r^2 \sin\theta\, d\theta\, d\phi = 4\pi r^2 E$$

Gauss Law $\rightarrow$ $4\pi r^2 E = \frac{4\pi\rho}{3\varepsilon}(r^3 - a^3) \rightarrow E = \frac{\rho}{3\varepsilon}\left(r - \frac{a^3}{r^2}\right)$

$$\boxed{\vec{E} = E\,\vec{e_r} = \frac{\rho}{3\varepsilon}\left(r - \frac{a^3}{r^2}\right)\vec{e_r} \qquad a < r < b}$$

## Region III

$$Q=\int_V \rho_v\,dv = \int_{r=a}^{b}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \rho r^2 \sin\theta\, dr\, d\theta\, d\phi = \rho \frac{r^3}{3}\Big|_{r=a}^{b} -\cos\theta\Big|_{\theta=0}^{\pi} \phi\Big|_{\phi=0}^{2\pi}$$

$$Q=\frac{4\pi\rho}{3}(b^3-a^3)$$

$$\oint \vec{E}\cdot\vec{ds} = \oint (E\vec{e_r})\cdot(r^2\sin\theta\, d\theta\, d\phi\, \vec{e_r}) = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} E r^2 \sin\theta\, d\theta\, d\phi$$

$$\oint \vec{E}\cdot\vec{ds} = 4\pi r^2 E$$

Gauss Law $\rightarrow$ $4\pi r^2 E = \dfrac{4\pi\rho}{3\varepsilon}(b^3-a^3)$

$$E=\frac{\rho}{3\varepsilon r^2}(b^3-a^3)$$

$$\boxed{\vec{E}=E\vec{e_r}=\frac{\rho}{3\varepsilon r^2}(b^3-a^3)\,\vec{e_r} \qquad r>b}$$